MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

9 MARCH 2019
9.00 am – 11.00 am
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 8 printed pages (inclusive of the front page) with 4 questions only. Pages 6 to 8 are appendices for the rules of inference, logical equivalence laws and Standard Normal distribution table.
2. Attempt **ALL FOUR** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.
4. Show **ALL** of your working steps clearly.

QUESTION 1 (10 MARKS)

a) Let $A = \{1,2,3,4,5,6\}$, $B = \{1,2,5,6,7\}$ and $C = \{2,6,8,10,12\}$.

i) Find $(A \Delta B) \cap C$. [1 mark]

ii) Find $A \cap C$ and $P(A \cap C)$. [2 marks]

b) Given R and S be relations on $A = \{1, 2, 3, 4\}$ that
$R = \{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4),(4,4)\}$ and $S = \{(1,1),(1,3),(3,4)\}$.

i) Is R reflexive? Justify your answer. [1 mark]

ii) Is R symmetric? Justify your answer. [1 mark]

iii) Is R transitive? Justify your answer. [1 mark]

iv) Find $S \circ R$ and $S \circ S$. [2 marks]

c) Let $R = \{(x, y) \mid x^2 > 2y\}$ be the relation on the set of positive integers.

i) List at least FOUR elements of R. [1 mark]

ii) Is R a function? Justify your answer. [1 mark]

Continued ...

QUESTION 2 (15 MARKS)

a) Construct a **truth table** to determine whether $[(\neg p \vee q) \leftrightarrow r] \rightarrow (q \vee r)$ is a tautology, contradiction or contingency. [4 marks]

b) Given the following argument:

"Either I will not learn MySQL or I will apply for an apprenticeship."
"If I buy a notebook then I will learn MySQL."
"Either I join the e-sports club or I buy a notebook."
"If I join the e-sport club then I will go to Oregon."
"I do not apply for an apprenticeship."
Therefore, "I will go to Oregon."

Let p, q, r, s and t represent the following statements:

p: I will learn MySQL.
q: I will apply for an apprenticeship.
r: I buy a notebook.
s: I join the e-sports club.
t: I will go to Oregon.

i) Translate the given argument into logical expressions. [3 marks]

ii) Show that the argument is valid with formal reasoning. State clearly each of the law that you used in your proof. [3 marks]

c) Given the recursion:

Initial value : $a_1 = \frac{1}{2}$,

Recurrence relation : $a_n = 5a_{n-1}$ for integer $n>1$.

i) Use the recursion to find a_3. [1 mark]

ii) Use induction method to prove that $a_n = \frac{1}{2}(5)^{n-1}$ for $n \geq 1$ is the explicit formula for the given recursion. [4 marks]

Continued ...

QUESTION 3 (12 MARKS)

a) Use the definition of big-Ω notation to show that $4x^3 - x$ is $\Omega(x^3)$. [1.5 marks]

b) In a group G of 7 students, each of the student is exactly a friend of n other students.

i) If n is either 2 or 3, find n and explain your answer using the concepts from graph theory. [2 marks]

ii) Draw a graph which represents the friendship relations of G using the found value of n (from part *b(i)*). [1 mark]

c) Consider the graph G, shown in *Figure 1*:

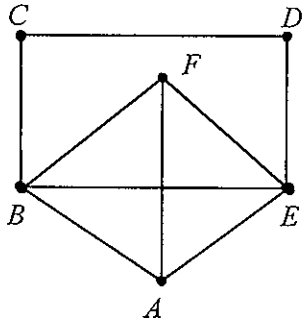

Figure 1

i) Determine whether G has an Euler cycle. Explain your answer. [1 mark]

ii) Write the definition of a Hamiltonian cycle and provide an example of a Hamiltonian cycle in G. [1.5 marks]

d) Consider the finite-state automaton M, shown in *Figure 2*:

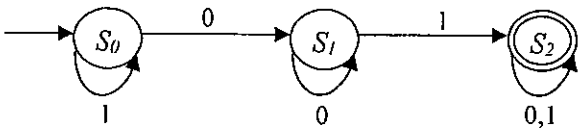

Figure 2

i) What are the initial state and accepting state(s) of M? [1 mark]

ii) Provide a next-state table for M. [3 marks]

iii) Use regular expression to express the language that is accepted by the automaton M. [1 mark]

Continued ...

QUESTION 4 (13 MARKS)

a) A student has to answer 8 out of 10 questions in an examination. How many choices does the student have if the first 3 questions must be answered? [3 marks]

b) A coin is loaded so that the probability of tails is 0.7. Suppose that the coin is tossed ten times and that the results of the tosses are mutually independent. What is the probability of obtaining at least two heads? **[Leave your answer correct to four decimal places.]** [3 marks]

c) The mean length of 500 tables produced in a factory is 150.5 cm and the standard deviation is 15 cm. Assuming the lengths are normally distributed, find how many tables with the length

i) between 119 and 155 cm. [4 marks]

ii) more than 185 cm. [3 marks]

Continued ...

Appendix I

List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Conversion of Equivalence: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Double Negation: $\neg \neg p \Leftrightarrow p$

DeMorgan :
(i) $\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$
(ii) $\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination :
(i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Negation :
(i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

Identity :
(i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Commutative :
(i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Idempotent :
(i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Distributive :
(i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Associative :
(i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$
(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

Absorption :
(i) $p \vee (p \wedge q) \Leftrightarrow p$
(ii) $p \wedge (p \vee q) \Leftrightarrow p$

Continued ...

Appendix II Standard Normal Curve Areas

Standard Normal Probabilities

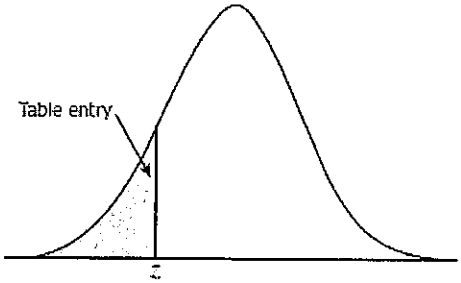

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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Standard Normal Probabilities

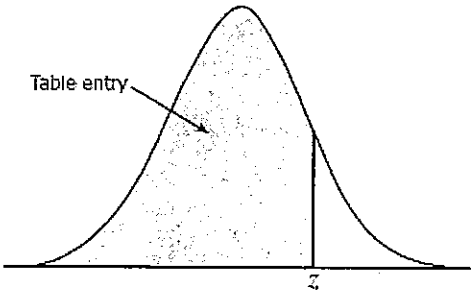

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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